Topics in Geometry: Geometric Group Theory

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Geometric group theory is the study of finitely-presented groups from a geometric viewpoint. Geometric group theorists study how the algebraic properties of groups are connected to the geometric properties of their Cayley graphs and of the spaces that they act on.

This course is an introduction to some of the basic ideas of geometric group theory. One of the main focuses will be the geometry of spaces and groups with negative and non-positive curvature and how to use this to understand groups that act on non-positively curved spaces.

Some of the topics I plan to address:

- Basics of geometric group theory: The "geometry" of a group is a little ill-defined, because different sets of generators correspond to different Cayley graphs and different word metrics. How can we define properties that don't depend on the choice of generators?

- Hyperbolic spaces and groups: Much of modern geometric group theory originated in Gromov's study of hyperbolic groups, which are a coarse version of negatively-curved spaces. Part of the power of this notion lies in the many equivalent definitions of hyperbolicity, and part of the power lies in how common these groups are (for example, Gromov proposed that there's a sense in which a "random" finitely presented group is hyperbolic). We will explore some of these definitions and their applications.

- The geometry of the word problem: The word problem is a classical and fundamental algorithmic problem about finitely-presented groups – given a product of generators, how do you identify what group element it represents? How do you tell if it's the identity? It turns out that you can interpret this combinatorial problem in a very geometric way, and we'll consider what this tells us about the geometry of various groups.

- Non-positively curved groups and spaces: How can we weaken the notion of hyperbolicity to include non-positively curved spaces? Subgroups of non-positively curved groups often have remarkable geometric properties and we'll study some examples.